$$\begin{aligned} \mathbf{H}_0 &\equiv \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^T \\ &\simeq [\mathbf{U}_R \;\; \mathbf{U}_D] \begin{bmatrix} \boldsymbol{\Sigma}_R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R^T \\ \mathbf{V}_D^T \end{bmatrix} \\ &= \mathbf{U}_R\,\boldsymbol{\Sigma}_R^{1/2}\,\boldsymbol{\Sigma}_R^{1/2}\,\mathbf{V}_R^T \end{aligned} \tag{13}$$

where $R \equiv rank(\mathbf{H}_0)$. Finally, a *balanced* realization of the system under question is obtained by pseudo-inverting various submatrix components (block 108 of FIGURE 1) as follows:

$$\mathbf{D} = \mathbf{Y}^0 \quad (N_o \times 1) \tag{14}$$

$$\mathbf{C} \simeq \mathbf{U}_R\,\boldsymbol{\Sigma}_R^{1/2} \quad (N_o \times R) \tag{15}$$

$$\mathbf{B} \simeq \text{the first } N_i \text{ columns of } \boldsymbol{\Sigma}_R^{1/2}\,\mathbf{V}_R^T \quad (R \times N_i) \tag{16}$$

$$\mathbf{A} \simeq \boldsymbol{\Sigma}_R^{-1/2}\,\mathbf{U}_R^T\,\mathbf{H}_1\,\mathbf{V}_R\,\boldsymbol{\Sigma}_R^{-1/2} \quad (R \times R) \tag{17}$$

Since $R << L$, the above model represents a reduced-order realization of the original system. Note that the realization is optimal in that it is balanced between *inputs* and *outputs*. However, the total number of samples taken is N_i x $(M+ 1)$, which increases proportional to the number of inputs. Also, for an accurate system identification $\mathbf{H}_0$ must have sufficient columns and rows, i.e., N_i x $(M- 1)$, $\geq R$ and $N_0 \geq R$.

For a very large data set $\mathbf{Y}^n$ with many time steps and a large number of inputs, the Eigensystem Realization Algorithm/Data Correlations (ERA/DC) method may be preferred. As described, for example, in Juang, J.N., *Applied System Identification,* Prentice Hall Englewood Cliffs, New Jersey, 1994, the ERA/DC method may be used to compress the amount of data and reduce the computation time required for the SVD of the Hankel matrix.

Although desirable results have been achieved using the prior art computational methods, there is room for improvement. For example, if the unsteady CFD model is driven

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With reference to FIGURE 2, the SCI/ERA method 200 proceeds as follows. First, at a block 202, individual pulse responses are sampled for the first two time steps:

$$\mathbf{Y}^0 = [\mathbf{y}_1^0 \ \mathbf{y}_2^0 \ \ldots \ \mathbf{y}_{N_i}^0] \tag{18}$$

$$\mathbf{Y}^1 = [\mathbf{y}_1^1 \ \mathbf{y}_2^1 \ \ldots \ \mathbf{y}_{N_i}^1] \tag{19}$$

Next, at a block 204, an SCI is constructed as follows:

$$\mathbf{b}_{SCI}^n \equiv \sum_{i=1}^{N_i} \mathbf{b}_i r_i^n \quad \text{(for states)} \tag{20}$$

$$\mathbf{d}_{SCI}^n \equiv \sum_{i=1}^{N_i} \mathbf{d}_i r_i^n \quad \text{(for outputs)} \tag{21}$$

where

$$r_i^n \equiv \text{a sequence of arbitrary numbers} \tag{22}$$

To ensure independency of the inputs, it is desirable that the signals be as uncorrelated as possible. In an ideal case they would be statistically uncorrelated random signals, i.e., $C_{ij}(m) = E[r^n_i \ r^{n-m}_i] = 0$ for $i \neq j$, but they are hard to construct for numerical analysis.

Subject to the SCI, at a block 206, the SCI/ERA method 200 samples the system response $\mathbf{y}^n$ for $n = 0, 1, 2, \ldots, M$, to obtain:

$$\mathbf{y}_{c0}^n \equiv \mathbf{C}\mathbf{x}^n = \mathbf{y}^n - \sum_{i=1}^{N_i} \mathbf{y}_i^0 r_i^n \tag{23}$$

$$\mathbf{y}_{c1}^n \equiv \mathbf{C}\mathbf{A}\mathbf{x}^n = \mathbf{y}^{n+1} - \sum_{i=1}^{N_i} \mathbf{y}_i^0 r_i^{n+1} - \sum_{i=1}^{N_i} \mathbf{y}_i^1 r_i^n \tag{24}$$

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